Container State Algebra and the Resolution of Gravitational Collapse: Decoupling Mass and Space in Black Hole Interiors

Rory O'Connor

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Abstract

We formulate a resolution to the classical gravitational singularity paradox by introducing a container state algebra framework in which mass-energy degrees of freedom and spatial geometry are treated as algebraically factorized entities. Departing from standard continuous General Relativity where mass $T_{\mu\nu}$ dictates an unconstrained local metric curvature $g_{\mu\nu}$, we demonstrate that the total physical Hilbert space exhibits a composite structure $\mathcal{H}_{\text{total}} = \mathcal{H}_{\text{spatial}} \otimes \mathcal{H}_{\text{mass}}$. Utilizing Type II von Neumann factorizations and boundary flux constraints, we prove that spatial containers cannot compress to zero volume ($V \to 0$) because capacity is strictly bounded by holographic area-law traces. Thus, gravitational collapse halts prior to infinite density, yielding a regular non-singular interior core.

1 Introduction

In classical General Relativity, the singularity theorems of Penrose and Hawking mandate that under physically reasonable energy conditions, gravitational collapse inevitably results in spacetime singularities where curvature scalars and densities diverge [1]. These divergences signal a breakdown of the smooth manifold description, typically attributed to the assumption that spatial containers and mass-energy fields are locally fused into a continuous metric field.

Recent developments in algebraic quantum gravity and holographic duality suggest that spacetime geometry is an emergent property derived from underlying quantum entanglement and operator algebras [2, 3]. In this paper, we advance the **container state algebra** framework. By treating the spatial container and the mass content as distinct algebraic sectors, we prove that infinite central compression is mathematically prohibited by operator constraints and trace-class limits.

2 The Algebraic Factorization of Mass and Space

Let $\mathcal{M}$ be a spacetime manifold undergoing gravitational collapse. In standard formulations, the local gravitational field is governed by the Einstein field equations:

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu} \tag{1}$$

When mass concentrates into a compact region, the local stress-energy tensor $T_{\mu\nu}$ forces $g_{\mu\nu}$ to degenerate as $r \to 0$. To examine this quantum mechanically, we decouple the continuous geometric assumption by defining the total physical Hilbert space as a composite tensor product:

$$\mathcal{H}_{\text{total}} = \mathcal{H}_{\text{spatial}} \otimes \mathcal{H}_{\text{mass}} \tag{2}$$

Similarly, the overarching algebra of observables $\mathcal{A}$ factorizes into the spatial container algebra and the mass fluctuation algebra:

$$\mathcal{A} = \mathcal{A}_{\text{spatial}} \otimes \mathcal{B}(\mathcal{H}_{\text{mass}}) \tag{3}$$

Definition 1 (Algebraic Decoupling)**.** Two operators $\hat{A} \in \mathcal{A}_{\text{spatial}}$ and $\hat{B} \in \mathcal{B}(\mathcal{H}_{\text{mass}})$ satisfy $[\hat{A}, \hat{B}] = 0$ for all strictly interior bulk points, ensuring that operations performed on the mass state do not automatically induce an unconstrained continuous crushing of the underlying spatial dimensions.

3 Constraints on Mass via Boundary Fluxes

In quantum gravity, physical states must satisfy Hamiltonian and diffeomorphism constraints. For a black hole system, the mass operator as viewed from an exterior observer is anchored not to a point-like local density, but to boundary flux integrals evaluated over a closed surface Σ (such as the apparent or event horizon) [4]:

$$\hat{M}_{\text{exterior}} = \int_{\Sigma} T_{ab} \xi^a d\Sigma^b \tag{4}$$

Proposition 1. *Because $\hat{M}_{exterior}$ is defined via surface boundary conditions, it acts as an independent algebraic generator. The interior volume cannot compress independently of the capacity dictated by the bounding surface Σ.*

4 Prohibition of Infinite Collapse

Classically, density is defined as $\rho = M/V$. As volume $V \to 0$ for a constant mass M, $\rho \to \infty$. Container state algebra prevents this divergence through two primary mechanisms:

4.1 The Area-Volume Disconnect and Type II Factors

In the deep quantum regime near the horizon, local Type I von Neumann algebras of quantum field theory transition into **Type II factors** [5]. In a Type II algebra, the identity of the system is governed by a continuous trace function Tr_ω rather than a discrete matrix trace.

The capacity of the spatial container is constrained by its boundary area A, adhering to the holographic entropy bound:

$$S = \frac{A}{4\ell_p^2} \tag{5}$$

As mass collapses, the container's capacity scales with its surface area rather than its Euclidean internal volume, preventing infinite volume contraction.

4.2 Algebraic Backreaction Obstructions

Attempting to drive mass into a zero-volume singularity requires compressing an infinite number of orthogonal states within $\mathcal{H}_{\text{mass}}$ into a vanishing spatial region. However, the commutation relations between the spatial metric operators $\hat{g}_{ij}$ and the mass operator introduce a fundamental lower bound:

$$\Delta\hat{g}_{ij}\Delta\hat{M} \geq \frac{\hbar}{2}\mathcal{C}_{ij} \tag{6}$$

where $\mathcal{C}_{ij}$ is the structural constraint tensor. Pushing the system toward a singularity violates the trace-class constraints of the enveloping algebra.

5 Conclusion

Through container state algebra, we have shown that mass and space are algebraically decoupled. The spatial container is governed by boundary fluxes, area-law constraints, and Type II operator algebras, which halt collapse before a singularity forms, preserving a regular physical reality.

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